

Miyabi Math Unit 05: Word Problems: Speed, Percent and Mixtures

Free Sample

Selected pages from the Student Edition

Word Problems: Speed, Percent and Mixtures — Step by Step

This is a free sample of *Word Problems: Speed, Percent and Mixtures — Step by Step (Unit 5)*. It contains selected pages from the Student Edition: key words, the four steps for word problems, and explanations from Parts 1, 2, 4 and 5, three worked examples (EX-03, EX-08, EX-10) and three practice questions (PR-05, PR-24, PR-33) with their answers. The full product includes 68 questions and a Solutions Edition with worked solutions for every question.

Part 1 · Units and distance = rate × time

Key words

- A **rate** tells you how much of something happens in one unit of time.
- **Speed** is the rate of travel: the distance traveled in one unit of time. In this unit, every speed is **constant** (it does not change) unless the problem says it changes.
- **km/h** is read "kilometers per hour." **m/min** is read "meters per minute." **m/s** is read "meters per second." Here km = kilometers, m = meters, h = hours, min = minutes and s = seconds.
- The main rule of this unit is

distance = rate × time, or **d = r × t**

We can also write $r = d \div t$ and $t = d \div r$.

- **Units must match.** If the rate is in km/h, the time must be in hours, and the distance is in kilometers. If the rate is in m/min, the time must be in minutes, and the distance is in meters.
- A **table** helps you organize a problem. Each row is one part of the trip:

	Rate	× Time	= Distance
part of the trip	r	t	$r \times t$

How to solve word problems (four steps)

1. **Find** — What does the question ask for? Write the quantity and its unit.
2. **Name** — Choose a letter for the unknown. Write "Let t = the time in hours."
3. **Equation** — Write an equation from the story. Then solve it.
4. **Check in the story** — Put your answer back into the **words** of the story, not only into your equation. Check every condition in the story.

How to write answers

- Write the answer as a sentence with units. Example: **Answer: The car travels 180 km.**
 - If a time in hours is a decimal, also give it in minutes. Example: **1.5 hours (90 minutes).**
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Part 2 · Speed problems with one equation

Key words

- Two objects move **toward each other** when they start at two places and move closer. If they start at the same time, the two distances add up to the starting gap when they meet.
- Two objects move in **opposite directions** when they start at the same place and move apart. The distance between them is the sum of their two distances.
- One object **catches up** with another when both move in the **same direction** from the same place and the faster one reaches the slower one. At that moment, the two distances are equal.
- Time = distance ÷ rate**. We can write $d \div 12$ as the fraction **d/12**. To clear fractions in an equation, multiply both sides by a number that every denominator divides into.

Worked examples

EX-03 Two cars start from the same place at the same time and drive in opposite directions on a straight road. One car drives at 70 km/h and the other at 50 km/h. After how many hours are the cars 300 km apart?

	Rate	× Time	= Distance
first car	70	t	70t
second car	50	t	50t

- Find: the time, in hours.
- Name: Let t = the time in hours.
- Equation: the two distances add up to 300 km.

$$\begin{array}{rcl}
 70t + 50t & = & 300 \\
 120t & = & 300 \quad \text{Combine the t-terms.} \\
 t & = & 2.5 \quad \text{Divide both sides by 120.}
 \end{array}$$

Answer: The cars are 300 km apart after 2.5 hours (2 hours 30 minutes).

Check in the story: $70 \times 2.5 = 175$ km; $50 \times 2.5 = 125$ km; $175 + 125 = 300$ km ✓

Practice B

Solve each problem in four steps: Find, Name, Equation, Check in the story. Write the answer as a sentence with units.

ID	Question
PR-05	Two cyclists start at the same time from two towns 70 km apart. They ride toward each other on the same road, one at 16 km/h and the other at 12 km/h. After how many hours do they meet? <i>Hint: The two distances add up to 70 km.</i>

Part 4 · Percent: increase, decrease and the original amount

Key words

- **Percent** means "per hundred." The symbol % means "out of 100." So $p\% = p/100$. For example, $15\% = 0.15$.
- **p% of a quantity** = $(p/100) \times$ the quantity. For example, 30% of $250\text{ kg} = 0.3 \times 250\text{ kg} = 75\text{ kg}$.
- The **original amount** is the amount before a change. We think of it as 100% .
- An **increase of p%**: new amount = original amount $\times (1 + p/100)$. For example, an increase of 15% means "multiply by 1.15 ."
- A **decrease of p%**: new amount = original amount $\times (1 - p/100)$. For example, a decrease of 20% means "multiply by 0.8 ."
- **Percent change** = $(\text{change} \div \text{original amount}) \times 100\%$.
- **Clearing decimals**. To solve an equation such as $0.8x = 360$, you can multiply both sides by 10 (or 100) so that every coefficient is an integer: $8x = 3,600$.
- In this unit, L = liters, mL = milliliters ($1\text{ L} = 1,000\text{ mL}$) and kg = kilograms.

Worked examples

EX-08 After a decrease of 20% , a water tank holds 360 L . How many liters did the tank hold before the decrease?

- Find: the original amount, in liters.
- Name: Let x = the amount before the decrease, in liters.
- Equation: a decrease of 20% means "multiply by 0.8 ."

$0.8x = 360$	
$8x = 3,600$	Multiply both sides by 10 .
$x = 450$	Divide both sides by 8 .

Answer: The tank held 450 L before the decrease.

Check in the story: 20% of $450\text{ L} = 90\text{ L}$; $450 - 90 = 360\text{ L}$ ✓

Watch out: A percent change is found by dividing by the **original** amount, not by the new amount.

Watch out: An increase of 20% followed by a decrease of 20% does not give back the starting amount. For example, $100 \rightarrow 120 \rightarrow 96$.

Practice D

Solve each problem in four steps: Find, Name, Equation, Check in the story. Write the answer as a sentence with units.

ID	Question
PR-24	A new bottle holds 25% more than an old bottle. The new bottle holds 1,500 mL. How many milliliters does the old bottle hold?

Part 5 · Concentration and mixtures

Key words

- A **salt solution** is salt dissolved in water. A **sugar solution** is sugar dissolved in water.
- The **mass of the solution** = the mass of the salt (or sugar) + the mass of the water. In this unit, every mass is in grams (g).
- The **concentration** of a solution is the percent of its mass that is salt (or sugar):

$$\text{concentration} = \text{mass of salt} \div \text{mass of solution} \times 100\%$$

So **mass of salt** = **concentration** $\times$ **mass of solution**. For example, 300 g of an 8% salt solution contains $0.08 \times 300 = 24$ g of salt.

- When **water is added** or **water is removed** (for example, when water **evaporates**, or turns into vapor and leaves), the mass of the salt stays the same. Only the mass of the solution changes.
- A **mixture** is made by mixing two or more solutions. The salt in the mixture = the sum of the salt in each solution. The mass of the mixture = the sum of the masses.
- A table helps:

	Mass of solution	$\times$ Concentration	= Mass of salt
solution	m	c	$c \times m$

Worked examples

EX-10 A 5% salt solution and a 10% salt solution are mixed to make 600 g of an 8% salt solution. How many grams of each solution are used?

	Mass of solution	$\times$ Concentration	= Mass of salt
5% solution	x	0.05	$0.05x$
10% solution	y	0.10	$0.10y$
mixture	600	0.08	48

- Find: the mass of each solution, in grams.
- Name: Let x = the mass of the 5% solution in grams. Let y = the mass of the 10% solution in grams.
- Equation:

Eq. 1:	$x + y = 600$	total mass
Eq. 2:	$0.05x + 0.10y = 48$	total salt ($0.08 \times 600 = 48$)
Eq. 3:	$5x + 10y = 4,800$	Multiply both sides of Eq. 2 by 100.
Eq. 4:	$5x + 5y = 3,000$	Multiply both sides of Eq. 1 by 5.
	$5y = 1,800$	Subtract Eq. 4 from Eq. 3.
	$y = 360$	
	$x + 360 = 600$	Substitute $y = 360$ into Eq. 1.
	$x = 240$	

Answer: 240 g of the 5% solution and 360 g of the 10% solution are used.

Check in the story: total mass $240 + 360 = 600$ g ✓; salt $0.05 \times 240 + 0.10 \times 360 = 12 + 36 = 48$ g, and $48 \div 600 = 0.08 = 8\%$ ✓

Watch out: The mass of the solution includes the salt. If 20 g of salt is dissolved in 180 g of water, the solution has a mass of 200 g, not 180 g.

Watch out: Concentrations are not added. Mixing some 4% solution with some 9% solution gives a concentration between 4% and 9%, not 13%.

Practice E

Solve each problem in four steps: Find, Name, Equation, Check in the story. Write the answer as a sentence with units.

ID	Question
PR-33	A person mixes 300 g of a 4% salt solution with 200 g of a 9% salt solution. What is the concentration of the mixture?

Answers

Answers only. Full solutions and checks: Solutions Edition.

Practice

ID	Answer
PR-05	They meet after 2.5 hours (2 hours 30 minutes).
PR-24	The old bottle holds 1,200 mL.
PR-33	The concentration of the mixture is 6%.