

Miyabi Math Unit 04:

Quadratic Equations: Basics

Free Sample

Selected pages from the Student Edition

Quadratic Equations: Basics

This is a free sample of *Quadratic Equations: Basics (Unit 4)*. It contains selected pages from the Student Edition: what this unit does not cover, and key words and explanations from Parts 1, 2, 3 and 6, three worked examples (EX-03, EX-06, EX-12) and three practice questions (PR-05, PR-17, PR-36) with their answers. The full product includes 68 questions and a Solutions Edition with worked solutions for every question.

What this unit does not cover

- the quadratic formula
 - equations whose solutions are not integers or fractions (for example, $x^2 = 2$)
 - equations with no solution among the numbers used in this book (for example, $x^2 = -4$)
 - equations that you would need to factor with an x^2 coefficient other than 1, such as $2x^2 + 7x + 3 = 0$
 - graphs of quadratic equations
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Part 1 · Quadratic equations and their solutions

Key words

- **quadratic equation** — an equation in which the highest power of x is x^2 . Examples: $x^2 - 5x + 6 = 0$, $x^2 = 49$, $(x + 1)(x - 4) = 14$.
- **solution** — a value of x that makes the equation true (the same meaning as in Unit 1).

A linear equation such as $3x - 5 = 16$ has one solution. A quadratic equation often has **two** solutions. To check a value, put it into the equation and see whether the left side and the right side are equal.

How answers are written in this unit

- Two solutions: **Answer: $x = 2$ or $x = 3$** . The word "or" means that each value makes the equation true on its own. The order does not matter; in this book the smaller value comes first.
 - One solution (Part 5): **Answer: $x = 5$ (one solution)**.
 - Each check is written on its own line: **Check $x = 2$: $2^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$ ✓**
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Part 2 · Solving with square roots

Key words

- **square root** — a square root of a number k is a number whose square is k . For example, 7 and -7 are both square roots of 49, because $7^2 = 49$ and $(-7)^2 = 49$.

If $x^2 = k$ and k is a positive number, then x is one of the two square roots of k : one is positive and the other is negative. So an equation $x^2 = k$ with k positive has two solutions. In this unit, k is always the square of an integer or a fraction, such as $49 = 7^2$ or $9/4 = (3/2)^2$.

The square of a number is never negative, so an equation such as $x^2 = -4$ has no solution among the numbers used in this book. Equations like this are not in this unit.

Worked examples

EX-03 Solve $x^2 = 49$.

$$\begin{array}{l} x^2 = 49 \\ x = 7 \text{ or } x = -7 \end{array} \quad \text{the two square roots of 49}$$

Answer: $x = -7$ or $x = 7$

$$\begin{array}{l} \text{Check } x = -7: \quad (-7)^2 = 49 \quad \checkmark \\ \text{Check } x = 7: \quad 7^2 = 49 \quad \checkmark \end{array}$$

Watch out: In equations such as $x^2 = 49$, both the positive and the negative square root are solutions. If you write only $x = 7$, one solution is missing.

Practice B · square roots

Solve. Check each solution.

PR-05 $x^2 = 64$

Hint: There are two numbers whose square is 64.

Part 3 · The zero product rule

Key words

- **zero product rule** — if $A \times B = 0$, then $A = 0$ or $B = 0$. (If two numbers multiply to 0, at least one of them is 0.)

To use the rule, the equation must have **0 on one side** and a **product** (factors multiplied together) on the other side. So:

1. Make one side 0.
2. Factor the other side (Unit 3).
3. Set each factor equal to 0 and solve.
4. Check each solution in the original equation.

Worked examples

EX-06 Solve $x^2 - 3x - 28 = 0$.

Find two integers whose product is -28 and whose sum is -3 : they are 4 and -7 .

$$\begin{array}{rcl}
 x^2 - 3x - 28 & = & 0 \\
 (x + 4)(x - 7) & = & 0 \qquad \text{factor} \\
 x + 4 = 0 \quad \text{or} \quad x - 7 = 0 & & \text{zero product rule} \\
 x = -4 \quad \text{or} \quad x = 7
 \end{array}$$

Answer: $x = -4$ or $x = 7$

$$\begin{array}{l}
 \text{Check } x = -4: \quad (-4)^2 - 3 \times (-4) - 28 = 16 + 12 - 28 = 0 \quad \checkmark \\
 \text{Check } x = 7: \quad 7^2 - 3 \times 7 - 28 = 49 - 21 - 28 = 0 \quad \checkmark
 \end{array}$$

Practice C · the zero product rule

Solve. Factor first if needed, then use the zero product rule. Check each solution.

PR-17 $x^2 + 3x - 40 = 0$

Hint: Find two integers whose product is -40 and whose sum is 3.

Part 6 · Word problems

Key words

- **consecutive integers** — integers that come one after the other, such as 4 and 5, or -3 and -2 . If the smaller one is x , the larger one is $x + 1$.

Four steps

1. **Find** — what does the question ask for?
2. **Name** — write "Let $x = \dots$ " for the unknown quantity, with its unit.
3. **Equation** — write the equation and solve it.
4. **Check in the story** — keep only the solutions that fit the story (for example, a length must be a positive number), and check each condition.

Worked examples

EX-12 A rectangle is 4 cm longer than it is wide. Its area is 60 square centimeters. Find the width.

Find: the width, in centimeters
 Name: Let $x =$ the width in cm. The length is $x + 4$ cm.
 Equation: $x(x + 4) = 60$ area = width $\times$ length
 $x^2 + 4x - 60 = 0$ expand, subtract 60
 $(x - 6)(x + 10) = 0$ factor
 $x = 6$ or $x = -10$ zero product rule
 A width cannot be negative, so $x = 6$.

Answer: The width is 6 cm.

Check in the story: length $6 + 4 = 10$ cm, which is 4 cm more than the width $\checkmark$; area $6 \times 10 = 60 \checkmark$

Watch out: The equation may have a solution that does not fit the story. Check each solution against the conditions in the problem before you write the answer.

Practice F · word problems

Solve. Use the four steps: Find, Name, Equation, Check in the story.

PR-36 A square has an area of 121 square meters. Find the length of one side.

Hint: A length is a positive number.

Answers

Answers only. Worked solutions and checks are in the Solutions Edition.

Practice

- PR-05: $x = -8$ or $x = 8$
- PR-17: $x = -8$ or $x = 5$
- PR-36: The length of one side is 11 m.