

# **Miyabi Math Unit 03: Expanding and Factoring**

---

Free Sample

Selected pages from the Student Edition

# Expanding and Factoring — Step by Step

This is a free sample of *Expanding and Factoring — Step by Step (Unit 3)*. It contains selected pages from the Student Edition: key words and explanations from Parts 1, 2, 4 and 5, three worked examples (EX-03, EX-07, EX-10) and three practice questions (PR-05, PR-22, PR-31) with their answers. The full product includes 68 questions and a Solutions Edition with worked solutions for every question.

---

## Part 1 · Terms, $x^2$ and like terms

---

### Key words

- A **term** is one part of an expression. Terms are joined by + or -. The expression  $3x^2 - 2x + 4$  has three terms:  $3x^2$ ,  $-2x$  and 4. We keep each sign with its term.
- The **coefficient** of a term is the number in front of the letter. In  $3x^2$ , the coefficient is 3. In  $-2x$ , the coefficient is  $-2$ . In  $x^2$ , the coefficient is 1 (we do not write the 1).
- A **constant term** is a term with no  $x$ . In  $3x^2 - 2x + 4$ , the constant term is 4.
- $x^2$  is read "x squared." It means  $x \times x$ . For example, when  $x = -5$ ,  $x^2 = (-5) \times (-5) = 25$ .
- **Like terms** have the same letter part.  $3x^2$  and  $-x^2$  are like terms.  $5x$  and  $-2x$  are like terms. But  $x^2$  and  $x$  are **not** like terms, so we cannot combine them.
- To **expand** means to multiply out the parentheses and write the result as terms joined by + or -. You did this in Unit 1, Part 5.

### How to write answers

- Write the  $x^2$  term first, then the  $x$  term, then the constant term. Example: **Answer:  $3x^2 - 2x + 4$**

### How to check

- **Check with a number.** Put the same number in place of  $x$  in the question and in your answer. We usually use  $x = 2$ . The two values must be the same. If they are different, there is a mistake.
  - If  $x = 2$  makes a pair of parentheses equal to 0, use  $x = 3$  instead. ( $0 = 0$  does not help you find mistakes.)
  - The same value is a good sign, but it does not show that the answer is right for **every** number. From Part 4, you will also **check by expanding**, which is the full check.
-

## Part 2 · Expanding $(x + a)(x + b)$

To expand two pairs of parentheses, multiply **each term** in the first pair by **each term** in the second pair. This gives 4 products. Then combine like terms.

### Worked examples

**EX-03** Expand  $(x + 2)(x + 5)$ .

$$\begin{aligned}
 &(x + 2)(x + 5) \\
 &= x \times x + x \times 5 + 2 \times x + 2 \times 5 && \text{Multiply } x \text{ by } x \text{ and by } 5. \text{ Then multiply } 2 \text{ by } x \text{ and by } 5. \\
 &= x^2 + 5x + 2x + 10 \\
 &= x^2 + 7x + 10 && 5x + 2x = 7x
 \end{aligned}$$

**Answer:**  $x^2 + 7x + 10$

Check with  $x = 2$ :  $(x + 2)(x + 5) = 4 \times 7 = 28$ ;  $x^2 + 7x + 10 = 4 + 14 + 10 = 28 \checkmark$

### Practice B — expanding $(x + a)(x + b)$

| ID    | Question                  | Hint                                                           |
|-------|---------------------------|----------------------------------------------------------------|
| PR-05 | Expand $(x + 3)(x + 4)$ . | Multiply $x$ by $x$ and by 4. Then multiply 3 by $x$ and by 4. |

## Part 4 · Taking out a common factor

### Key words

- A **factor** is one part of a product. In  $3 \times (2x + 5)$ , the factors are 3 and  $(2x + 5)$ .
- To **factor** an expression means to write it as a product. Factoring is the **reverse** of expanding.
- A **common factor** is a factor that **every** term has. In  $6x + 15$ , every term can be divided by 3, so 3 is a common factor.

### How to take out a common factor

1. Find the **largest** number that divides every coefficient (and the constant term).
2. If **every** term has  $x$ , then  $x$  is also part of the common factor.
3. Write the common factor in front. Inside the parentheses, write what is left of each term.
4. **Check by expanding.** Expanding your answer must give the expression in the question.

The order of factors does not matter:  $3(2x + 5)$  and  $(2x + 5) \times 3$  are equal. In this book, we write the common factor first.

### Worked examples

**EX-07** Factor  $6x + 15$ .

$$\begin{aligned} 6x + 15 &= 3 \times 2x + 3 \times 5 && 3 \text{ is the largest number that divides both 6 and 15.} \\ &= 3(2x + 5) && \text{Take out the common factor 3.} \end{aligned}$$

**Answer:**  $3(2x + 5)$

Check by expanding:  $3(2x + 5) = 3 \times 2x + 3 \times 5 = 6x + 15 \checkmark$

Check with  $x = 2$ :  $6x + 15 = 12 + 15 = 27$ ;  $3(2x + 5) = 3 \times 9 = 27 \checkmark$

### Practice D — taking out a common factor

Take out the largest common factor.

| ID    | Question           | Hint                                                |
|-------|--------------------|-----------------------------------------------------|
| PR-22 | Factor $8x - 12$ . | The largest number that divides both 8 and 12 is 4. |

## Part 5 · Factoring $x^2 + bx + c$

In Part 2, you saw that  $(x + m)(x + n) = x^2 + (m + n)x + mn$ .

So, to factor  $x^2 + bx + c$ , look for **two numbers** that:

- **multiply** to  $c$  (the constant term), and
- **add** to  $b$  (the coefficient of  $x$ ).

### Signs of the two numbers

| c        | b                    | The two numbers               |
|----------|----------------------|-------------------------------|
| positive | positive             | both positive                 |
| positive | negative             | both negative                 |
| negative | positive or negative | one positive and one negative |

### Worked examples

**EX-10** Factor  $x^2 - 2x - 15$ .

We need two numbers that multiply to  $-15$  and add to  $-2$ .

$c = -15$  is negative, so one number is positive and one is negative.

Pairs that multiply to  $-15$       Sum

1 and  $-15$      $-14$

$-1$  and  $15$      $14$

3 and  $-5$      $-2$     ✓

$-3$  and  $5$      $2$

$$x^2 - 2x - 15 = (x + 3)(x - 5)$$

**Answer:**  $(x + 3)(x - 5)$

Check by expanding:  $(x + 3)(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$  ✓

Check with  $x = 2$ :  $x^2 - 2x - 15 = 4 - 4 - 15 = -15$ ;  $(x + 3)(x - 5) = 5 \times (-3) = -15$  ✓

**Watch out:**  $(x - 3)(x + 5)$  looks similar, but it expands to  $x^2 + 2x - 15$ . The signs matter. Always check by expanding.

### Practice E — factoring $x^2 + bx + c$

| ID    | Question                 | Hint                                                                                            |
|-------|--------------------------|-------------------------------------------------------------------------------------------------|
| PR-31 | Factor $x^2 - 8x + 12$ . | $c$ is positive and $b$ is negative, so both numbers are negative. For the check, use $x = 3$ . |

## Answers

---

Answers only. For every step and check, see the Solutions Edition. For factoring, the factors may be written in a different order. For example,  $(x + 5)(x + 3)$  is the same answer as  $(x + 3)(x + 5)$ .

### Practice

| ID    | Answer           |
|-------|------------------|
| PR-05 | $x^2 + 7x + 12$  |
| PR-22 | $4(2x - 3)$      |
| PR-31 | $(x - 2)(x - 6)$ |